Determine if each series below is convergent or divergent. **Do NOT find the sum.** **SCORE: _____ / 16 PTS**

[a] $\sum_{n=1}^{\infty} \frac{1}{1+e^{-n}}$ SUBTOTAL = 3

(1) $\lim_{n\to\infty} \frac{1}{1+e^{-n}} = \frac{1}{1+0} = 1 \neq 0$ (1)

SERIES DIVERGES

(1)

[b] $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ SUBTOTAL = 8

COMPARE TO $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n^2} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

(2) $\lim_{n\to\infty} \frac{\frac{\sqrt{n+2}}{2n^2+n+1}}{\frac{1}{2n^{\frac{3}{2}}}}$

$= \lim_{n\to\infty} \frac{2n^{\frac{3}{2}}\sqrt{n+2}}{2n^2+n+1} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

(1) $= \lim_{n\to\infty} \frac{2\sqrt{1+\frac{2}{n}}}{2+\frac{1}{n}+\frac{1}{n^2}} = \frac{2\sqrt{1+0}}{2+0+0} = 1 \neq 0$ (1) (1)

(1) $\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ CONVERGES $(p = \frac{3}{2} > 1)$

SO $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ CONVERGES (1)

(1)

SUBTOTAL = 5

[c] $\sum_{n=1}^{\infty} \frac{1}{ne^n}$ (1)

$0 < \frac{1}{ne^n} \leq \frac{1}{e^n}$ FOR $n \geq 1$ (1)

(1) $\sum_{n=1}^{\infty} \frac{1}{e^n}$ CONVERGES $(r = \frac{1}{e} < 1)$ (1)

SO $\sum_{n=1}^{\infty} \frac{1}{ne^n}$ CONVERGES

(1)

Consider the following statements. **SCORE: _____ / 3 PTS**

(i) If the terms of $\{a_n\}$ and $\{b_n\}$ are positive, and $\lim_{n\to\infty} \frac{a_n}{b_n} \neq 0$, and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent

(ii) If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} b_n$ is divergent

(iii) If $a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent

Which of the statements above are true ? Circle the correct answer below.

[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

It is true that $\int_1^{\infty} e^{-\pi x}\cos \pi x \, dx$ converges. A student writes that this means that $\sum_{n=1}^{\infty} e^{-\pi n}\cos \pi n$ converges according to the integral test. Is the student's reasoning correct ? Why or why not ?

SCORE: _____ / 2 PTS

$\frac{1}{2}$ INCORRECT, $e^{-\pi x}\cos \pi x$ OSCILLATES (INCREASES + DECREASES)

$1\frac{1}{2}$ IF YOU GOT EITHER ANSWER/REASON

AND IS NOT ALWAYS POSITIVE (WHEN n IS ODD)

Determine if $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ is convergent or divergent, **without using the comparison and limit comparison tests.**

SCORE: _____ / 9 PTS

$f(x) = \frac{e^{\frac{1}{x}}}{x^2} > 0$ $\frac{1}{2}$, CONTINUOUS $\frac{1}{2}$ ON $[1, \infty)$

$f'(x) = \frac{e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)x^2 - 2x e^{\frac{1}{x}}}{(x^2)^2} = -\frac{e^{\frac{1}{x}}(1+2x)}{x^4}$ 2 < 0 $\frac{1}{2}$ ON $[1, \infty)$

SO $\frac{1}{2}$ $f(x)$ IS DECREASING ON $[1, \infty)$

$$\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \int -e^u\,du = -e^u + C = -e^{\frac{1}{x}} + C$$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$$\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx = \lim_{N\to\infty} -e^{\frac{1}{x}}\Big|_1^N = \lim_{N\to\infty}\left(-e^{\frac{1}{N}} + e\right) = -e^0 + e = e - 1$$

2 1

CONVERGES 1

SO $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ CONVERGES 1